

# Leading finite-size effects on some three-point correlators in $AdS_5 \times S^5$

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## Abstract

In the framework of the semiclassical approach, we find the leading finite-size effects on the normalized structure constants in some three-point correlation functions in  $AdS_5 \times S^5$ , expressed in terms of the conserved string angular momenta  $J_1$ ,  $J_2$ , and the worldsheet momentum  $p_w$ , identified with the momentum  $p$  of the magnon excitations in the dual spin-chain arising in  $\mathcal{N} = 4$  SYM in four dimensions.

# 1 Introduction

The correspondence between type IIB string theory on  $\text{AdS}_5 \times S^5$  target space and the  $\mathcal{N} = 4$  super Yang-Mills theory (SYM) in four space-time dimensions, in the planar limit, is the most studied example of the AdS/CFT duality [1]. A lot of impressive progresses have been made in this field of research based on the integrability structures discovered on both sides of the correspondence (for recent overview on AdS/CFT integrability, see [2]).

Various classical string solutions play an important role in testing and understanding the AdS/CFT correspondence. To establish relations with the dual gauge theory, we have to take the semiclassical limit of *large* conserved charges like string energy  $E$  and spins  $S_{1,2}$  on  $\text{AdS}_5$  and angular momenta  $J_{1,2,3}$  on  $S^5$  [3].

An example of such string solution is the so called "giant magnon", for which the energy  $E$  and the angular momentum  $J_1$  go to infinity, but the difference  $E - J_1$  is finite, while  $S_{1,2} = 0$ ,  $J_{2,3} = 0$  [4]. It lives on  $R_t \times S^2$  subspace of  $\text{AdS}_5 \times S^5$ , and gave a strong support for the conjectured all-loop  $SU(2)$  spin chain, arising in the dual  $\mathcal{N} = 4$  SYM, and made it possible to get a deep insight in the AdS/CFT duality. This was extended to the giant magnon bound state ( $J_2 \neq 0$ ), or *dyonic* giant magnon, corresponding to a string moving on  $R_t \times S^3$  and related to the complex sine-Gordon model [5]. Further extension to  $R_t \times S^5$  have been also worked out in [6], where it was also shown that such type of string solutions can be obtained by reduction of the string dynamics to the Neumann-Rosochatius integrable system. It can be used also for studding the *finite-size effects*, related to the wrapping interactions in the dual field theory [7]. From the string theory viewpoint, the leading and even sub-leading finite size effect on the giant magnon dispersion relation was first found and described in [8]. The case of leading finite-size effect on *dyonic* giant magnon dispersion relation was considered in [9]. There, the string theory result was compared with the result coming from the  $\mu$ -term Lüscher correction, based on the  $S$ -matrix description. Both results coincide.

During the years, many important achievements concerning correlation functions in the AdS/CFT context have been made. Recently, interesting developments have been done by considering general heavy string states [10]-[70]<sup>1</sup>.

In [37, 39], the three-point correlation functions of finite-size (dyonic) giant magnons [4, 5] and three different "light" states have been obtained. They are given in terms of hypergeometric functions and several parameters. However, it is important to know their dependence on the conserved string charges  $J_1$ ,  $J_2$  and the worldsheet momentum  $p$ , because namely these quantities are related to the corresponding operators in the dual gauge theory, and the momentum of the magnon excitations in the dual spin-chain. That is why, we are going to find this dependence here. Unfortunately, this can not be done exactly for the finite-size case due to the complicated dependence between the above mentioned parameters and  $J_1$ ,  $J_2$ ,  $p$ . Because of that, we will consider only the leading order finite-size effects on

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<sup>1</sup>Some papers devoted to the field theory side of the problem are also included here.

the three-point correlators. In this paper, we will restrict ourselves to the case of  $AdS_5 \times S^5 / \mathcal{N} = 4$  SYM duality.

The paper is organized as follows. In Sec. 2, we first give a short review of the giant magnon solution. Then, we explain the limitations under which the three-point correlation functions considered here are computed and give the exact results in the semiclassical limit. Sec. 3 is devoted to the computation of the leading order finite-size effects on the three-point correlators given in Sec. 2 in terms of the conserved string angular momenta and the worldsheet momentum  $p$ . In Sec. 4 we conclude with some final remarks.

## 2 Finite-size giant magnons and three-point correlators

### 2.1 Review of the giant magnon solutions

We denote with  $Y, X$  the coordinates in  $AdS_5$  and  $S^5$  parts of the background  $AdS_5 \times S^5$ .

$$\begin{aligned} Y_1 + iY_2 &= \sinh \rho \sin \eta e^{i\varphi_1}, \\ Y_3 + iY_4 &= \sinh \rho \cos \eta e^{i\varphi_2}, \\ Y_5 + iY_0 &= \cosh \rho e^{it}. \end{aligned}$$

The coordinates  $Y$  are related to the Poincare coordinates by

$$\begin{aligned} Y_m &= \frac{x_m}{z}, \\ Y_4 &= \frac{1}{2z} (x^m x_m + z^2 - 1), \\ Y_5 &= \frac{1}{2z} (x^m x_m + z^2 + 1), \end{aligned}$$

where  $x^m x_m = -x_0^2 + x_i x_i$ , with  $m = 0, 1, 2, 3$  and  $i = 1, 2, 3$ . We parameterize  $S^5$  as in [26].

Euclidean continuation of the time-like directions to  $t_e = it$ ,  $Y_{0e} = iY_0$ ,  $x_{0e} = ix_0$ , will allow the classical trajectories to approach the  $AdS_5$  boundary  $z = 0$  when  $\tau_e \rightarrow \pm\infty$ , and to compute the corresponding correlation functions.

The dyonic finite-size giant magnon solution, where  $(\tau, \sigma)$  are the world-sheet coordinates, can be written as ( $t = \sqrt{W}\tau$ ,  $i\tau = \tau_e$ )

$$x_{0e} = \tanh(\sqrt{W}\tau_e), \quad x_i = 0, \quad z = \frac{1}{\cosh(\sqrt{W}\tau_e)},$$

$$\begin{aligned}
\cos \theta &= \sqrt{\chi_p} \, dn \left( \frac{\sqrt{1-u^2}}{1-v^2} \sqrt{\chi_p} (\sigma - v\tau) \middle| 1-\epsilon \right), \\
\phi_1 &= \frac{\tau - v\sigma}{1-v^2} + \frac{vW}{\sqrt{1-u^2} \sqrt{\chi_p} (1-\chi_p)} \times \\
\Pi \left( -\frac{\chi_p}{1-\chi_p} (1-\epsilon), am \left( \frac{\sqrt{1-u^2}}{1-v^2} \sqrt{\chi_p} (\sigma - v\tau) \right) \middle| 1-\epsilon \right) \\
\phi_2 &= u \frac{\tau - v\sigma}{1-v^2},
\end{aligned} \tag{2.1}$$

where  $\theta$  is the angle on which the metric on  $S^3 \subset S^5$  depends, while  $\phi_{1,2}$  are the isometric angles on it.  $dn(\alpha|1-\epsilon)$  is one of the Jacobi elliptic functions,  $\Pi(\alpha, \beta|1-\epsilon)$  is the incomplete elliptic integral of third kind, and  $am(x)$  is the Jacobi amplitude. Let us also mention that  $\chi_p, \chi_m$  are related to  $u, v, W$  parameters according to

$$\begin{aligned}
\chi_p + \chi_m &= \frac{2 - (1+v^2)W - u^2}{1-u^2}, \\
\chi_p \chi_m &= \frac{1 - (1+v^2)W + (vW)^2}{1-u^2},
\end{aligned} \tag{2.2}$$

and

$$\epsilon \equiv \frac{\chi_m}{\chi_p}. \tag{2.3}$$

For the finite-size dyonic giant magnon string solution, the explicit expressions for the conserved quantities and the worldsheet momentum  $p$  can be written as [30]

$$\begin{aligned}
\mathcal{E} &= \frac{2\sqrt{W}(1-v^2)}{\sqrt{1-u^2} \sqrt{\chi_p}} \mathbf{K}(1-\epsilon), \\
\mathcal{J}_1 &= \frac{2\sqrt{\chi_p}}{\sqrt{1-u^2}} \left[ \frac{1-v^2W}{\chi_p} \mathbf{K}(1-\epsilon) - \mathbf{E}(1-\epsilon) \right], \\
\mathcal{J}_2 &= \frac{2u\sqrt{\chi_p}}{\sqrt{1-u^2}} \mathbf{E}(1-\epsilon)
\end{aligned} \tag{2.4}$$

$$p = \frac{2v}{\sqrt{1-u^2} \sqrt{\chi_p}} \left[ \frac{W}{1-\chi_p} \Pi \left( -\frac{\chi_p}{1-\chi_p} (1-\epsilon) \middle| 1-\epsilon \right) - \mathbf{K}(1-\epsilon) \right], \tag{2.5}$$

where<sup>2</sup>

$$\mathcal{E} = \frac{2\pi E}{\sqrt{\lambda}}, \quad \mathcal{J}_{1,2} = \frac{2\pi J_{1,2}}{\sqrt{\lambda}}$$

are the string energy and the two angular momenta.  $\mathbf{K}(1-\epsilon)$ ,  $\mathbf{E}(1-\epsilon)$ , and  $\Pi \left( -\frac{\chi_p}{1-\chi_p} (1-\epsilon) \middle| 1-\epsilon \right)$  are the complete elliptic integrals of first, second and third kind. As

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<sup>2</sup>The relation between the string tension  $T$  and the t'Hooft coupling  $\lambda$  in the dual  $\mathcal{N} = 4$  SYM is  $TR^2 = \sqrt{\lambda}/2\pi$ , where  $R$  is the common radius of  $AdS_5$  and  $S^5$  subspaces. Here  $R$  is set to 1.

explained in [8]<sup>3</sup>, (2.5) should be identified with the momentum of the magnon excitations in the spin chain arising in the dual  $\mathcal{N} = 4$  SYM theory.

The dyonic giant magnon dispersion relation, including the leading finite-size correction, can be written as

$$\mathcal{E} - \mathcal{J}_1 = \frac{\sqrt{\lambda}}{2\pi} \left[ \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} - \frac{\sin^4(p/2)}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \epsilon \right], \quad (2.6)$$

where

$$\epsilon = 16 \exp \left[ - \frac{2 \left( \mathcal{J}_1 + \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \sin^2(p/2)}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \right]. \quad (2.7)$$

The second term in (2.6) represents the leading finite-size effect on the energy-charge relation, which disappears for  $\epsilon \rightarrow 0$ , or equivalently  $\mathcal{J}_1 \rightarrow \infty$ . It is nonzero only for  $\mathcal{J}_1$  finite.

The above two equalities are found under the following conditions on the parameters

$$0 < u < 1, \quad 0 < v < 1, \quad 0 < W < 1, \quad 0 < \chi_m < \chi_p < 1.$$

The case of finite-size giant magnons with one angular momentum can be obtained by setting  $u = 0$ , or  $\mathcal{J}_2 = 0$ , as can be seen from (2.4).

## 2.2 Three-point correlation functions

It is known that the correlation functions of any conformal field theory can be determined in principle in terms of the basic conformal data  $\{\Delta_i, C_{ijk}\}$ , where  $\Delta_i$  are the conformal dimensions defined by the two-point correlation functions

$$\langle \mathcal{O}_i^\dagger(x_1) \mathcal{O}_j(x_2) \rangle = \frac{C_{12} \delta_{ij}}{|x_1 - x_2|^{2\Delta_i}}$$

and  $C_{ijk}$  are the structure constants in the operator product expansion

$$\langle \mathcal{O}_i(x_1) \mathcal{O}_j(x_2) \mathcal{O}_k(x_3) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} |x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2} |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1}}.$$

Therefore, the determination of the initial conformal data for a given conformal field theory is the most important step in the conformal bootstrap approach.

The three-point functions of two "heavy" operators and a "light" operator can be approximated by a supergravity vertex operator evaluated at the "heavy" classical string configuration [14, 26]:

$$\langle V_H(x_1) V_H(x_2) V_L(x_3) \rangle = V_L(x_3)_{\text{classical}}.$$

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<sup>3</sup>See also [9] for the dyonic case.

For  $|x_1| = |x_2| = 1$ ,  $x_3 = 0$ , the correlation function reduces to

$$\langle V_H(x_1)V_H(x_2)V_L(0) \rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.$$

Then, the normalized structure constants

$$\mathcal{C} = \frac{C_{123}}{C_{12}}$$

can be found from

$$\mathcal{C} = c_\Delta V_L(0)_{\text{classical}}, \quad (2.8)$$

where  $c_\Delta$  is the normalized constant of the corresponding "light" vertex operator.

Recently, first results describing *finite-size* effects on the three-point correlators appeared [30, 31, 35, 37, 39]. This was done for the cases when the "heavy" string states are *finite-size* giant magnons, carrying one or two angular momenta, and for three different choices of the "light" state:

1. Primary scalar operators:  $V_L = V_j^{pr}$
2. Dilaton operator:  $V_L = V_j^d$
3. Singlet scalar operators on higher string levels:  $V_L = V^q$

The corresponding (unintegrated) vertices are given by [14]

$$\begin{aligned} V_j^{pr} = & (Y_4 + Y_5)^{-\Delta_{pr}} (X_1 + iX_2)^j \\ & [z^{-2} (\partial x_m \bar{\partial} x^m - \partial z \bar{\partial} z) - \partial X_k \bar{\partial} X_k], \end{aligned} \quad (2.9)$$

where the scaling dimension is  $\Delta_{pr} = j$ . The corresponding operator in the dual gauge theory is  $Tr(Z^j)$ <sup>4</sup>.

$$\begin{aligned} V_j^d = & (Y_4 + Y_5)^{-\Delta_d} (X_1 + iX_2)^j \\ & [z^{-2} (\partial x_m \bar{\partial} x^m + \partial z \bar{\partial} z) + \partial X_k \bar{\partial} X_k], \end{aligned} \quad (2.10)$$

where now the scaling dimension  $\Delta_d = 4 + j$  to the leading order in the large  $\sqrt{\lambda}$  expansion. The corresponding operator in the dual gauge theory is proportional to  $Tr(F_{\mu\nu}^2 Z^j + \dots)$ , or for  $j = 0$ , just to the SYM Lagrangian.

$$V^q = (Y_4 + Y_5)^{-\Delta_q} (\partial X_k \bar{\partial} X_k)^q. \quad (2.11)$$

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<sup>4</sup> $Z$  is one of the three complex scalars contained in  $\mathcal{N} = 4$  SYM.

This operator corresponds to a scalar *string* state at level  $n = q - 1$ , and to leading order in  $\frac{1}{\sqrt{\lambda}}$  expansion

$$\Delta_q = 2 \left( \sqrt{(q-1)\sqrt{\lambda} + 1 - \frac{1}{2}q(q-1) + 1} \right). \quad (2.12)$$

The value  $n = 1 (q = 2)$  corresponds to a massive string state on the first excited level and the corresponding operator in the dual gauge theory is an operator contained within the Konishi multiplet. Higher values of  $n$  label higher string levels.

The results obtained for the normalized structure constants (2.8), for the case of finite-size giant magnons in  $AdS_5 \times S^5$ , and the above three vertices, are as follows [37, 39]

$$\begin{aligned} \mathcal{C}_j^{pr} = \pi^{3/2} c_j^{pr} \frac{\Gamma\left(\frac{j}{2}\right)}{\Gamma\left(\frac{3+j}{2}\right)} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \\ \left[ (1-W+j(1-v^2W)) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1-\epsilon\right) \right. \\ \left. - (1+j)(1-u^2)\chi_p {}_2F_1\left(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1-\epsilon\right) \right], \end{aligned} \quad (2.13)$$

$$\begin{aligned} \mathcal{C}_j^d = 2\pi^{3/2} c_{4+j}^d \frac{\Gamma\left(\frac{4+j}{2}\right)}{\Gamma\left(\frac{5+j}{2}\right)} \frac{\chi_p^{\frac{j-1}{2}}}{\sqrt{(1-u^2)W}} \\ \left[ (1-u^2)\chi_p {}_2F_1\left(\frac{1}{2}, -\frac{1}{2} - \frac{j}{2}; 1; 1-\epsilon\right) \right. \\ \left. - (1-W) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{j}{2}; 1; 1-\epsilon\right) \right], \end{aligned} \quad (2.14)$$

$$\begin{aligned} \mathcal{C}^q = c_{\Delta_q} \pi^{3/2} \frac{\Gamma\left(\frac{\Delta_q}{2}\right)}{\Gamma\left(\frac{\Delta_q+1}{2}\right)} \frac{(-1)^q [2 - (1+v^2)W]^q}{(1-v^2)^{q-1} \sqrt{(1-u^2)W} \chi_p} \\ \sum_{k=0}^q \frac{q!}{k!(q-k)!} \left[ -\frac{1-u^2}{1 - \frac{1}{2}(1+v^2)W} \right]^k \chi_p^k {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - k; 1; 1-\epsilon\right), \end{aligned} \quad (2.15)$$

where  ${}_2F_1(a, b; c; z)$  is Gauss' hypergeometric function.

### 3 Leading order finite-size effects

As we already point out in the beginning, (2.4), (2.5), can not be solved *exactly* with respect to the parameters involved, in order to express the relevant three-point correlation functions

in terms of the conserved charges and  $p$ . That is why, we will consider here only the leading order finite-size effects on the three-point correlators. This means that we will consider the limit  $\mathcal{J}_1$  large, i.e.  $J_1 \gg \sqrt{\lambda}$ , where the finite-size corrections to both conformal dimensions and energies of string states have been computed also from the Lüscher corrections. Practically, the problem reduces to consider the limit  $\epsilon \rightarrow 0$ , since  $\epsilon = 0$  corresponds to the infinite-size case, i.e.  $\mathcal{J}_1 = \infty$ . The relevant expansions of the parameters are [30]

$$\begin{aligned}\chi_p &= \chi_{p0} + (\chi_{p1} + \chi_{p2} \log(\epsilon)) \epsilon, & \chi_m &= \chi_{m1} \epsilon, & W &= 1 + W_1 \epsilon, \\ v &= v_0 + (v_1 + v_2 \log(\epsilon)) \epsilon, & u &= u_0 + (u_1 + u_2 \log(\epsilon)) \epsilon.\end{aligned}\tag{3.1}$$

The coefficients on the first line in (3.1) can be obtained by using the equalities (2.2) and the definition of  $\epsilon$  (2.3) to be

$$\begin{aligned}\chi_{p0} &= 1 - \frac{v_0^2}{1 - u_0^2}, \\ \chi_{p1} &= \frac{v_0}{(1 - v_0^2)(1 - u_0^2)^2} \left\{ v_0 [(1 - v_0^2)^2 - 3(1 - v_0^2)u_0^2 + 2u_0^4 - 2(1 - v_0^2)u_0u_1] \right. \\ &\quad \left. - 2(1 - v_0^2)(1 - u_0^2)v_1 \right\}, \\ \chi_{p2} &= -2v_0 \frac{v_2 + (v_0u_2 - u_0v_2)u_0}{(1 - u_0^2)^2}, \\ \chi_{m1} &= 1 - \frac{v_0^2}{1 - u_0^2}, \\ W_1 &= -\frac{(1 - u_0^2 - v_0^2)^2}{(1 - u_0^2)(1 - v_0^2)}.\end{aligned}\tag{3.2}$$

The coefficients in the expansions of  $v$  and  $u$ , we take from [71], where for the case under consideration we have to set  $K_1 = \chi_{n1} = 0$ , or equivalently  $\Phi = 0$ . This leads to

$$\begin{aligned}v_0 &= \frac{\sin(p)}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}}, & u_0 &= \frac{\mathcal{J}_2}{\sqrt{\mathcal{J}_2^2 + 4\sin^2(p/2)}} \\ v_1 &= \frac{v_0(1 - v_0^2 - u_0^2)}{4(1 - u_0^2)(1 - v_0^2)} [(1 - v_0^2)(1 - \log(16)) - u_0^2(5 - v_0^2(1 + \log(16)) - \log(4096))] \\ v_2 &= \frac{v_0(1 - v_0^2 - u_0^2)}{4(1 - u_0^2)(1 - v_0^2)} [1 - v_0^2 - u_0^2(3 + v_0^2)] \\ u_1 &= \frac{u_0(1 - v_0^2 - u_0^2)}{4(1 - v_0^2)} [1 - \log(16) - v_0^2(1 + \log(16))] \\ u_2 &= \frac{u_0(1 - v_0^2 - u_0^2)}{4(1 - v_0^2)} (1 + v_0^2).\end{aligned}\tag{3.3}$$

We need also the expression for  $\epsilon$ . It can be found from the expansion of  $\mathcal{J}_1$ , and to the leading order is given by (2.7).



### 3.1 Giant magnons and primary scalar operators

Let us first point out that (2.13) simplifies a lot when  $j$  is odd ( $j = 2m + 1, m = 0, 1, 2, \dots$ ). In that case, Gauss' hypergeometric functions in (2.13) reduce to polynomials. This results in

$$\begin{aligned} \mathcal{C}_{2m+1}^{pr} = & \pi^{3/2} c_{2m+1}^{pr} \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m + 2)} \frac{\epsilon^{m/2} \chi_p^m}{\sqrt{(1 - u^2)W}} \\ & \left[ -2(m + 1)(1 - u^2)\sqrt{\epsilon} \chi_p P_{m+1}\left(\frac{1 + \epsilon}{2\sqrt{\epsilon}}\right) \right. \\ & \left. + (1 - W + (2m + 1)(1 - v^2W)) P_m\left(\frac{1 + \epsilon}{2\sqrt{\epsilon}}\right) \right], \end{aligned} \quad (3.4)$$

where  $P_n(z)$  are Legendre's polynomials.

Since the corresponding operators in the dual gauge theory are of the type  $Tr(Z^j)$ , we will restrict ourselves to integer-valued  $j$ .

Let us start with the simpler case when  $J_2 = 0$ , or equivalently  $u = 0$ . Expanding (2.13) in  $\epsilon$  and using (3.1) - (3.3), one finds that<sup>5</sup>

$$\begin{aligned} \mathcal{C}_{10}^{pr} &\approx 0, \quad \mathcal{C}_{20}^{pr} \approx \frac{4}{3} c_2^{pr} \mathcal{J}_1 \sin^2(p/2) \epsilon, \\ \mathcal{C}_{j0}^{pr} &\approx c_j^{pr} a_j \sin(p/2)^{j+1} \epsilon, \quad j = 3, \dots, 10, \end{aligned} \quad (3.5)$$

where

$$\epsilon = 16 \exp[-2 - \mathcal{J}_1 \csc(p/2)], \quad (3.6)$$

for the case under consideration<sup>6</sup>. The numerical coefficients  $a_j$  are given by

$$a_j = \left( \frac{1}{4}\pi^2, \frac{2^4}{3.5}, \frac{1}{16}\pi^2, \frac{2^7}{3^2.5.7}, \frac{3.5}{2^9}\pi^2, \frac{2^{10}}{3^3.5^2.7}, \frac{5.7}{2^{11}}\pi^2, \frac{2^{14}}{3^2.5^2.7^2.11} \right).$$

A few comments are in order. From (3.5) one can conclude that the  $\mathcal{C}_{10}^{pr}$  and  $\mathcal{C}_{20}^{pr}$  cases are exceptional, while  $\mathcal{C}_{j0}^{pr}$  have the same structure for  $j \geq 3$ .  $\mathcal{C}_{10}^{pr} \approx 0$  means that the small  $\epsilon$  - contribution to the three point correlator is zero to the leading order in  $\epsilon$ .  $\mathcal{C}_{20}^{pr}$  is the only one normalized structure constant of this type proportional to  $\mathcal{J}_1$ . It is still exponentially suppressed by  $\epsilon$ . The common feature of  $\mathcal{C}_{j0}^{pr}$  in (3.5) is that they all vanish in the *infinite size* case, i.e., for  $\epsilon = 0$ . This property was established in [26], and confirmed even for the  $\gamma$ -deformed case in [37]. Here, we obtained the leading finite-size corrections to it.

Now, let us turn to the dyonic case, i.e.  $J_2 \neq 0$ . Working in the same way, but with  $u \neq 0$ , we derive

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<sup>5</sup>We use the notation  $\mathcal{C}_{j0}^{pr}$  in order to say that  $\mathcal{C}_j^{pr}$  are computed for the case  $J_2 = 0$ .

<sup>6</sup>This expression for  $\epsilon$  comes from (2.7) after setting  $\mathcal{J}_2 = 0$ .

$j = 1$ :

$$\begin{aligned}
\mathcal{C}_1^{pr} \approx & c_1^{pr} \frac{\pi^2}{16} \frac{\mathcal{J}_2^2 \csc(p/2)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^{3/2} [\mathcal{J}_2^2 + 4 \sin^4(p/2)]} \times \\
& \{ 8[\mathcal{J}_2^2 + 4 \sin^2(p/2)][\mathcal{J}_2^2 + 4 \sin^4(p/2)] \\
& + \sin^2(p/2) [40 + 17\mathcal{J}_2^2 + 2\mathcal{J}_2^4 - 20(3 + \mathcal{J}_2^2) \cos(p) \\
& + 3(8 + \mathcal{J}_2^2) \cos(2p) - 4 \cos(3p) - 4 \frac{\mathcal{J}_2^2 + 8 \sin^2(p/2)}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \times \\
& \left( \mathcal{J}_1 \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} + \mathcal{J}_2^2 + 4 \sin^2(p/2) \right) \times \\
& (\mathcal{J}_2^2 + 4 \sin^4(p/2) + 2 \sin^2(p)) \sin^2(p/2) \} \epsilon,
\end{aligned} \tag{3.7}$$

$j = 2$ :

$$\begin{aligned}
\mathcal{C}_2^{pr} \approx & \frac{4}{3} c_2^{pr} \frac{1}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^{3/2} [\mathcal{J}_2^2 + 4 \sin^4(p/2)]} \times \\
& \{ 2\mathcal{J}_2^2 [\mathcal{J}_2^2 + 4 \sin^2(p/2)] [\mathcal{J}_2^2 + 4 \sin^4(p/2)] - \sin^4(p/2) \times \\
& [20 + 3\mathcal{J}_2^2 - 2\mathcal{J}_2^4 - 2(15 + 2\mathcal{J}_2^2) \cos(p) + (12 + \mathcal{J}_2^2) \cos(2p) - 2 \cos(3p) \\
& + \frac{8}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \left( \mathcal{J}_1 \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} + \mathcal{J}_2^2 + 4 \sin^2(p/2) \right) \times \\
& (-3 + 2(2 + \mathcal{J}_2^2) \cos(p) - \cos(2p)) \sin^4(p/2) \} \epsilon,
\end{aligned} \tag{3.8}$$

$j = 3$ :

$$\begin{aligned}
\mathcal{C}_3^{pr} \approx & c_3^{pr} \frac{\pi^2}{256} \csc(p/2) \frac{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^{5/2}}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \times \\
& \left\{ 48\mathcal{J}_2^2 \sin^2(p/2) \frac{\mathcal{J}_2^2 + 4 \sin^4(p/2)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} - \left[ \frac{25\mathcal{J}_2^4}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^2} \right. \right. \\
& - \mathcal{J}_2^2 \frac{\mathcal{J}_2^2 + 4 \sin^4(p/2)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} (21 - 16 \cos(p) - 5 \cos(2p) + 8\mathcal{J}_2^2) \\
& - \frac{3}{2} \mathcal{J}_2^6 \frac{11 - 12 \cos(p) + \cos(2p) + 6\mathcal{J}_2^2}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4} \\
& + \left( 3\mathcal{J}_2^2 \left( \mathcal{J}_2^2 + 4 \sin^2(p/2) + \mathcal{J}_1 \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right) \times \right. \\
& (80 + 42\mathcal{J}_2^2 + 12\mathcal{J}_2^4 - (120 + 47\mathcal{J}_2^2 - 4\mathcal{J}_2^4) \cos(p) \\
& + (8 + \mathcal{J}_2^2) (6 \cos(2p) - \cos(3p))) \sin^4(p/2) \left. \frac{1}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4 [\mathcal{J}_2^2 + 4 \sin^4(p/2)]} \right. \\
& \left. - \frac{20\mathcal{J}_2^4 \sin^2(p)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} + \frac{3\mathcal{J}_2^4 \sin^4(p)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4} - 8 \left( \frac{\mathcal{J}_2^2 + 4 \sin^4(p/2)}{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right)^2 \right] \epsilon \},
\end{aligned} \tag{3.9}$$

$j = 4$ :

$$\begin{aligned}
c_4^{pr} \approx & \frac{2}{45} c_4^{pr} \frac{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^{5/2}}{\mathcal{J}_2^2 + 4 \sin^4(p/2)} \\
& \left\{ \frac{32 \mathcal{J}_2^2 [\mathcal{J}_2^2 + 4 \sin^4(p/2)] \sin^2(p/2)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} - \left[ \frac{17 \mathcal{J}_2^4}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^2} \right. \right. \\
& - \frac{1}{2} \mathcal{J}_2^2 \frac{\mathcal{J}_2^2 + 4 \sin^4(p/2)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} (39 - 32 \cos(p) - 7 \cos(2p) + 16 \mathcal{J}_2^2) \\
& - \mathcal{J}_2^6 \frac{11 - 12 \cos(p) + \cos(2p) + 6 \mathcal{J}_2^2}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4} \\
& + \left( 2 \mathcal{J}_2^2 \left( \mathcal{J}_2^2 + 4 \sin^2(p/2) + \mathcal{J}_1 \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right) \times \right. \\
& (75 + 44 \mathcal{J}_2^2 + 16 \mathcal{J}_2^4 - 2 (58 + 23 \mathcal{J}_2^2 - 4 \mathcal{J}_2^4) \cos(p) \\
& + 4 (13 + \mathcal{J}_2^2) \cos(2p) - 2 (6 + \mathcal{J}_2^2) \cos(3p) + \cos(4p)) \sin^4(p/2) \times \\
& \left. \frac{1}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4 [\mathcal{J}_2^2 + 4 \sin^4(p/2)]} \right. \\
& \left. - \frac{13 \mathcal{J}_2^4 \sin^2(p)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^3} + \frac{2 \mathcal{J}_2^4 \sin^4(p)}{[\mathcal{J}_2^2 + 4 \sin^2(p/2)]^4} - 3 \left( \frac{\mathcal{J}_2^2 + 4 \sin^4(p/2)}{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \right)^2 \right] \epsilon \Bigg\}.
\end{aligned} \tag{3.10}$$

In the four formulas above  $\epsilon$  is given by (2.7).

### 3.2 Giant magnons and dilaton operator

The leading finite-size effect on the normalized structure constant in the three-point correlator of two finite-size giant magnon's states and zero-momentum dilaton operator ( $j = 0$ ), in the limit  $J_1 \gg \sqrt{\lambda}$ , has been considered in [30]. Here, we will deal with the  $j > 0$  cases. Since the corresponding operators in the dual gauge theory are proportional to  $Tr (F_{\mu\nu}^2 Z^j + \dots)$ , we will restrict ourselves to integer-valued  $j$ .

When  $j$  is odd ( $j = 2m + 1, m = 0, 1, 2, \dots$ ), the normalized structure constants (2.14) simplify to

$$\begin{aligned}
c_{2m+1}^d = & 2\pi^{3/2} c_{2m+5}^d \frac{\Gamma(m + \frac{5}{2})}{\Gamma(m + 3)} \frac{\epsilon^{m/2} \chi_p^m}{\sqrt{(1-u^2)W}} \\
& \left[ (1-u^2)\sqrt{\epsilon} \chi_p P_{m+1} \left( \frac{1+\epsilon}{2\sqrt{\epsilon}} \right) - (1-W) P_m \left( \frac{1+\epsilon}{2\sqrt{\epsilon}} \right) \right].
\end{aligned} \tag{3.11}$$

Expanding (2.14) in  $\epsilon$  and using (3.1) - (3.3), one finds

$j = 1$ :

$$\begin{aligned} \mathcal{C}_1^d \approx & \frac{3}{4} \pi^2 c_5^d \sin^3(p/2) \left\{ \frac{1}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \right. \\ & - \frac{1}{128 (\mathcal{J}_2^2 + 4 \sin^2(p/2))^{3/2} (\mathcal{J}_2^2 + 4 \sin^4(p/2))^2} \left[ \left( 840 + 826 \mathcal{J}_2^2 + 258 \mathcal{J}_2^4 - 24 \mathcal{J}_2^6 \right. \right. \\ & - 2 (744 + 707 \mathcal{J}_2^2 + 244 \mathcal{J}_2^4 + 72 \mathcal{J}_2^6) \cos(p) \\ & + 4 (255 + 218 \mathcal{J}_2^2 + 62 \mathcal{J}_2^4 - 6 \mathcal{J}_2^6) \cos(2p) - (520 + 367 \mathcal{J}_2^2 + 24 \mathcal{J}_2^4) \cos(3p) \\ & + 2 (92 + 47 \mathcal{J}_2^2 + 3 \mathcal{J}_2^4) \cos(4p) - (40 + 11 \mathcal{J}_2^2) \cos(5p) + 4 \cos(6p) \Big) \\ & + 8 \mathcal{J}_1 \sin^2(p/2) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \left( (8 + 19 \mathcal{J}_2^2 + 12 \mathcal{J}_2^4) \cos(p) + (8 - 16 \mathcal{J}_2^2) \cos(2p) \right. \\ & \left. \left. - (8 + 3 \mathcal{J}_2^2) \cos(3p) - 2 (5 + 5 \mathcal{J}_2^2 - 2 \mathcal{J}_2^4 - \cos(4p)) \right) \right] \epsilon \Big\}, \end{aligned}$$

$j = 2$ :

$$\begin{aligned} \mathcal{C}_2^d \approx & \frac{2^8}{325} c_6^d \sin^4(p/2) \left\{ \frac{1}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \right. \\ & - \frac{1}{128 (\mathcal{J}_2^2 + 4 \sin^2(p/2))^{3/2} (\mathcal{J}_2^2 + 4 \sin^4(p/2))^2} \left[ \left( 210 + 8 \mathcal{J}_2^2 (6 - \mathcal{J}_2^2) (7 + 4 \mathcal{J}_2^2) \right. \right. \\ & - 8 (63 + 84 \mathcal{J}_2^2 + 38 \mathcal{J}_2^4 + 16 \mathcal{J}_2^6) \cos(p) \\ & + (585 + 576 \mathcal{J}_2^2 + 176 \mathcal{J}_2^4 - 32 \mathcal{J}_2^6) \cos(2p) - 4 (115 + 84 \mathcal{J}_2^2 + 4 \mathcal{J}_2^4) \cos(3p) \\ & + 2 (111 + 56 \mathcal{J}_2^2 + 4 \mathcal{J}_2^4) \cos(4p) - 4 (15 + 4 \mathcal{J}_2^2) \cos(5p) + 7 \cos(6p) \Big) \\ & - 8 \mathcal{J}_1 \sin^2(p/2) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \left( 15 + 8 \mathcal{J}_2^2 - 8 \mathcal{J}_2^4 - 4 (3 + 5 \mathcal{J}_2^2 + 4 \mathcal{J}_2^4) \cos(p) \right. \\ & \left. \left. - (12 - 8 \mathcal{J}_2^2) \cos(2p) + 4 (3 + \mathcal{J}_2^2) \cos(3p) - 3 \cos(4p) \right) \right] \epsilon \Big\}, \end{aligned}$$

$j = 3$ :

$$\begin{aligned} \mathcal{C}_3^d \approx & \frac{3.5}{2^5} \pi^2 c_7^d \sin^5(p/2) \left\{ \frac{1}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \right. \\ & + \frac{1}{960 (\mathcal{J}_2^2 + 4 \sin^2(p/2))^{3/2} (\mathcal{J}_2^2 + 4 \sin^4(p/2))^2} \left[ 20 \left( 256 (13 + 15 \cos(p)) \sin^{10}(p/2) \right. \right. \\ & + 288 \mathcal{J}_2^2 (5 + 7 \cos(p)) \sin^8(p/2) + \mathcal{J}_2^4 (54 + 241 \cos(p) + 10 \cos(2p) \\ & + 15 \cos(3p)) \sin^2(p/2) + 10 \mathcal{J}_2^6 \cos(p) (5 + 3 \cos(p)) \left. \right) \\ & + 60 \mathcal{J}_1 \sin^2(p/2) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \left( 20 + 6 \mathcal{J}_2^2 - 12 \mathcal{J}_2^4 \right. \\ & - (16 + 21 \mathcal{J}_2^2 + 20 \mathcal{J}_2^4) \cos(p) - 2 (8 - 5 \mathcal{J}_2^2) \cos(2p) \\ & \left. \left. + (16 + 5 \mathcal{J}_2^2) \cos(3p) - 4 \cos(4p) \right) \right] \epsilon \left. \right\}, \end{aligned}$$

$j = 4$ :

$$\begin{aligned} \mathcal{C}_4^d \approx & \frac{2^{11}}{3 \cdot 5^2 \cdot 7} c_8^d \sin^6(p/2) \left\{ \frac{1}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)}} \right. \\ & + \frac{1}{8192 (\mathcal{J}_2^2 + 4 \sin^2(p/2))^{3/2} (\mathcal{J}_2^2 + 4 \sin^4(p/2))^2} \left[ 64 \left( 294 + 14 \mathcal{J}_2^2 - 60 \mathcal{J}_2^4 + 48 \mathcal{J}_2^6 \right. \right. \\ & - 4 (51 - 49 \mathcal{J}_2^2 - 53 \mathcal{J}_2^4 - 36 \mathcal{J}_2^6) \cos(p) \\ & - (435 + 8 \mathcal{J}_2^2 (61 + 19 \mathcal{J}_2^2 - 6 \mathcal{J}_2^4)) \cos(2p) + 2 (305 + 209 \mathcal{J}_2^2 + 6 \mathcal{J}_2^4) \cos(3p) \\ & - 2 (179 + 83 \mathcal{J}_2^2 + 6 \mathcal{J}_2^4) \cos(4p) + 2 (53 + 13 \mathcal{J}_2^2) \cos(5p) - 13 \cos(6p) \left. \right) \\ & + 512 \mathcal{J}_1 \sin^2(p/2) \sqrt{\mathcal{J}_2^2 + 4 \sin^2(p/2)} \left( 25 + 4 \mathcal{J}_2^2 - 16 \mathcal{J}_2^4 - (20 + 22 \mathcal{J}_2^2 + 24 \mathcal{J}_2^4) \cos(p) \right. \\ & \left. \left. - 4 (5 - 3 \mathcal{J}_2^2) \cos(2p) + 2 (10 + 3 \mathcal{J}_2^2) \cos(3p) - 5 \cos(4p) \right) \right] \epsilon \left. \right\}. \end{aligned}$$

In the four formulas above  $\epsilon$  is given by (2.7).

Actually, we computed the normalized coefficients in the three-point correlators up to  $j = 10$ . However, since the expressions for them are too complicated, we give here only the results for the first two odd and two even values of  $j$ . Knowing these expressions, the conclusion is that they have the same structure for any  $j$  in the small  $\epsilon$  limit<sup>7</sup>. Namely

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<sup>7</sup>The only difference in that sense is that for  $j$  odd an additional overall factor of  $\pi^2$  appears, as can be seen from the formulas above.

$$\mathcal{C}_j^d \approx A_j c_{j+4}^d \sin^{j+2} \left( \frac{p}{2} \right) \left\{ \frac{1}{\sqrt{\mathcal{J}_2^2 + 4 \sin^2 \left( \frac{p}{2} \right)}} + \frac{a_j}{(\mathcal{J}_2^2 + 4 \sin^2 \left( \frac{p}{2} \right))^{3/2} (\mathcal{J}_2^2 + 4 \sin^4 \left( \frac{p}{2} \right))^2} \right. \\ \left. \left[ P_j^3(\mathcal{J}_2^2) + \mathcal{J}_1 \sin^2 \left( \frac{p}{2} \right) \sqrt{\mathcal{J}_2^2 + 4 \sin^2 \left( \frac{p}{2} \right)} Q_j^2(\mathcal{J}_2^2) \right] \epsilon \right\}. \quad (3.12)$$

where  $\epsilon$  is given in (2.7),  $A_j$  and  $a_j$  are numerical coefficients, while  $P_j^3(\mathcal{J}_2^2)$  and  $Q_j^2(\mathcal{J}_2^2)$  are polynomials of third and second order respectively, with coefficients depending on  $p$  in a trigonometric way.

Now, let us restrict ourselves to the simpler case when  $\mathcal{J}_2 = 0$ , i.e. giant magnon string states with one (large) angular momentum  $\mathcal{J}_1 \neq 0$ . Knowing the above results for  $1 \leq j \leq 10$ , one can conclude that the normalized structure constants in the three-point correlators for any  $j \geq 1$  in the small  $\epsilon$  limit look like<sup>8</sup>

$$\mathcal{C}_{j0}^d \approx \frac{A_j}{2} c_{j+4}^d \sin^j \left( \frac{p}{2} \right) \left[ \sin \left( \frac{p}{2} \right) \right. \\ \left. + \left( B_{j0} \sin \left( \frac{p}{2} \right) + C_{j0} \sin \left( \frac{3p}{2} \right) + D_{j0} (1 + \cos(p)) \mathcal{J}_1 \right) e^{-2 - \frac{\mathcal{J}_1}{\sin \frac{p}{2}}} \right], \quad (3.13)$$

where

$$B_{j0} = (-2^2, 3, \frac{2.11}{3}, 11, \frac{2^3 3^2}{5}, \frac{53}{3}, \frac{2.73}{7}, 2^3 3, \dots) \quad \text{for } j = (1, \dots, 8, \dots), \\ C_{j0} = 1 + 3j, \quad D_{j0} = 2(j + 1).$$

### 3.3 Giant magnons and singlet scalar operators on higher string levels

For that case, the expressions for the normalized structure constants in the three-point correlation functions for *dyonic* giant magnons are too long and complicated. That is why, we will write down here the results for *finite-size* giant magnon states only, i.e. for  $\mathcal{J}_2 = 0$ . Then, after small  $\epsilon$  expansion, one can find that (2.15) reduces to<sup>9</sup>

$$\mathcal{C}_0^q \approx c_{\Delta_q} \frac{\sqrt{\pi}}{A q_0} \frac{\Gamma \left( \frac{\Delta_q}{2} \right)}{\Gamma \left( \frac{1 + \Delta_q}{2} \right)} \{ A q_1 \sin(p/2) + A q_2 \mathcal{J}_1 \\ + [(A q_3 + A q_4 \cos(p)) \sin(p/2) + (A q_5 + A q_6 \cos(p)) \mathcal{J}_1 \\ + A q_7 \csc(p/2) (1 + \cos(p)) \mathcal{J}_1^2] \epsilon \}, \quad (3.14)$$

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<sup>8</sup>  $\mathcal{C}_{j0}^d$  is used for  $\mathcal{C}_j^d$  computed for  $\mathcal{J}_2 = 0$  case.

<sup>9</sup>  $\mathcal{C}_0^q \equiv \mathcal{C}^q$  computed for  $\mathcal{J}_2 = 0$ .

where  $Aq_i$  ( $i = 0, 1, \dots, 7$ ) are numerical coefficients, and for the case at hand  $\epsilon$  is given by (3.6).

This is the general structure of  $\mathcal{C}_0^q$ . The values of  $Aq_i$  we found are as follows ( $q = 1, \dots, 10$ )

$$\begin{aligned}
Aq_0 &= (8, 24, 60, 420, 2520, 27720, 180180, 180180, 3063060, 116396280), \\
Aq_1 &= (16, -16, 152, -632, 7216, -55216, 559304, -420312, 10089896, -301915216), \\
Aq_2 &= (-8, 24, -60, 420, -2520, 27720, -180180, 180180, -3063060, 116396280), \\
Aq_3 &= (2, -66, 147, -2575, 13446, -272694, 1555993, -2484923, 37469109, -2088496586), \\
Aq_4 &= (2, -10, 171, -1027, 15334, -144942, 1747825, -1523631, 41620821, -1396357874), \\
Aq_5 &= \left( -5, 31, -\frac{187}{2}, \frac{1837}{2}, -6343, 86653, -\frac{1256569}{2}, \frac{3}{2}490499, -\frac{27342361}{2}, 587890603 \right), \\
Aq_6 &= \left( 1, 13, -\frac{97}{2}, \frac{1207}{2}, -4453, 65863, -\frac{986299}{2}, \frac{3}{2}400409, -\frac{22747771}{2}, 500593393 \right), \\
Aq_7 &= \left( -1, 3, -\frac{15}{2}, \frac{105}{2}, -315, 3465, -\frac{45045}{2}, \frac{3}{2}15015, -\frac{765765}{2}, 14549535 \right).
\end{aligned} \tag{3.15}$$

## 4 Concluding Remarks

In this paper, in the framework of the semiclassical approach, we computed the leading finite-size effects on the normalized structure constants in some three-point correlation functions in  $AdS_5 \times S^5$ , expressed in terms of the conserved string angular momenta  $J_1$ ,  $J_2$ , and the worldsheet momentum  $p_w$ , identified with the momentum  $p$  of the magnon excitations in the dual spin-chain arising in  $\mathcal{N} = 4$  SYM in four dimensions. Namely, we found the leading finite-size effects on the structure constants in three-point correlators of two "heavy" (dyonic) giant magnon's string states and the following three "light" states:

1. Primary scalar operators;
2. Dilaton operator with nonzero-momentum ( $j \geq 1$ );
3. Singlet scalar operators on higher string levels.

A natural generalization of the above results would be to consider the case of  $\gamma$ -deformed (or  $TsT$ -transformed)  $AdS_5 \times S^5$  type IIB string theory background. Another possible issue to investigate is the case of  $AdS_4 \times CP^3$  type IIA string theory background, dual to  $\mathcal{N} = 6$  super Chern-Simons-matter theory in three space-time dimensions (ABJM model) and its  $TsT$ -deformations. We hope to report on these soon.

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